A NUMERICAL SOLUTION FOR ADDRESSING OVERTURNING PHENOMENA OF HERITAGE ASSETS

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ABSTRACT

Historical heritage represent a crucial aspect for societies and therefore it should be preserved from natural disasters such as earthquake. Base isolation systems are widely used to mitigate the horizontal effects of strong ground motions on important buildings and bridges, but there are also interesting applications on statues. However, such systems are characterized by properties that are quite different from the ones that belong to traditional civil structures. For this reason, national and international regulations are not exhaustive and actual dynamics of the system should be studied through numerical and experimental methods. Starting from analytical formulations, the paper investigates the sliding and rocking motion in details, being the typical one of statues under seismic loads. The presented numerical model describes the problem and is an alternative to the analytical formulation to perform several analyses automatically. In addition, it allows running parametric analyses to assess the influence of various parameters, such as eccentricity, stiffness, mass, geometric ratios, etc. Future work is geared to validate the numerical model trough performing experimental tests on shaking table.

Introduction to Rigid Body's free motion

Heritage structures play a significant testimony of the social culture of each country and it deserved to be preserved. The modeling of sliding and rocking behavior of an anchored rigid body under the earthquake was a challenging topic for last decades. The first real attempt at developing a mathematical model of the rigid body motion was made by Housner [1]. Afterwards, several studies have been made to improve the mathematical model considering different motions. The motion of a rigid un-anchored body generally is divided to six conditions including rest, slide, rock, slide-rock, free flight and impact. Several studies have been carried out to evaluate the different aspects of these motions ([2], [3] and [4]). The classical rocking motion equation of a free rigid block under an earthquake is given as [1]:

$$(1+mR^2)\ddot{\theta} = m\ddot{x}_g R\cos(\theta_c - |\theta|) - \text{sgn}(\theta) mRg\sin(\theta c - |\theta|) \tag{1}$$

where $\ddot{x}_g$ is the horizontal ground acceleration, θ is the rotation angle of the rigid body under

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excitation, m is the mass of the rigid block, R is the distance between center of gravity (CG) and overturning point, θ_c is the angle formed by R and the vertical direction. Fig. 1 shows the schematic rocking motion and geometrical parameters of a rigid body under horizontal seismic acceleration.

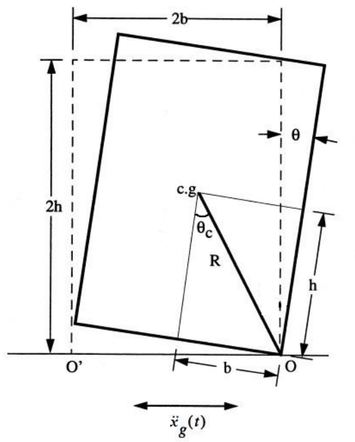


Figure 1. Schematic rocking motion of a 2D rigid block subjected to ground acceleration.

Although the later works ([5], [6] and [7]) are much closer to the reality, but still there are many grey areas to predict the exact transition between the different motion phases and the understanding of rigid bodies dynamic remains a research topic open to new contributions. In addition, they mostly do not include the eccentricities and consider the center of gravity equally distant from the two base corners. This paper tackles to develop an exhaustive method that can evaluate the different phases of motion for a 3D rigid body under a tri-directional earthquake (horizontals and vertical components).

Numerical Modeling

The problem has been modeled using an idealized 3-D model representative of the dynamic behavior of the statues under the earthquake. The rigid body has been modeled by using three rigid beams. Two rigid perpendicular beams have been considered to model the contact surface, and the other beam element is used to identify the position of CG above the contact surface. By defining different lengths of rigid beams the model is able to take into account the eccentricity as a variable. Fig. 2 shows the model schematically.

To predict accurately the sliding and rocking behavior of the statue under the earthquake, the computer program SAP2000 has been used in this study. This software is capable to model the geometric nonlinearity of the contact surfaces. Therefore to consider the effect of friction, a Friction-Pendulum Isolator element has been used to model the contact surface. This element is able to model combination of different conditions vary from at-rest to slide, or from uplift to slam-down for the cases of friction and rocking, respectively. The pendulum radii of the slipping surface has been set to zero to consider the flat surface friction. The element models the coupled biaxial friction at contact surface considering the post-slip stiffness. The friction forces are proportional to both external normal force and friction coefficient. The axial force (P) is modeled with a compression-only gap element that does not carry the tension force in the case of uplift and it is given by:

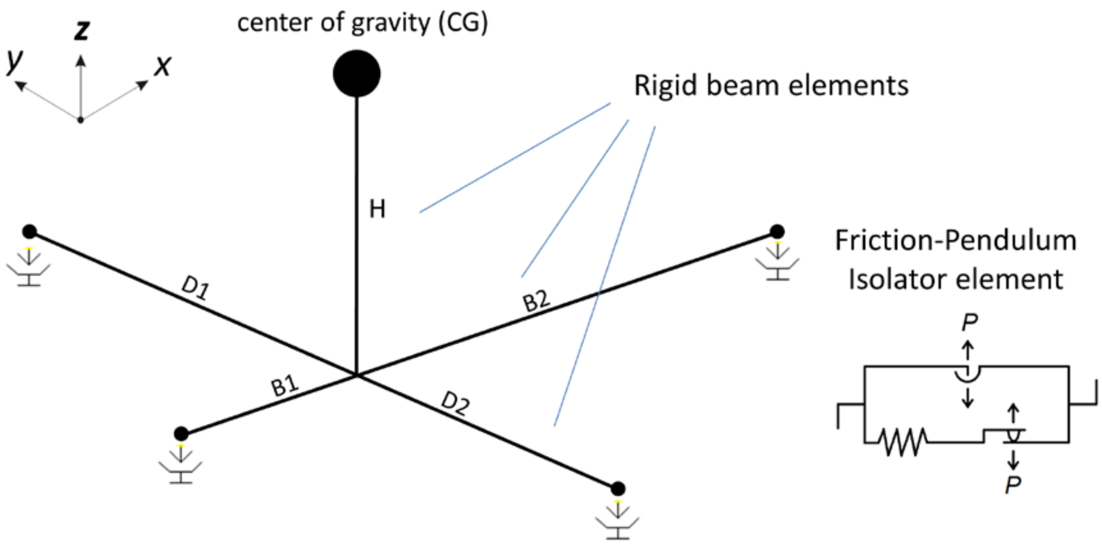


Figure 2. Mathematical modeling to evaluate the sliding and rocking behavior using the Friction-Pendulum Isolator element.

$$P = \begin{cases} K_z d_z & \text{if } d_z < 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where the K_z is the vertical stiffness in negative axial direction (-Z) and d_z is the vertical displacement of the rigid body base at the contact surface. K_z has been set to some large value in order to consider the rigidity of the contact surface. The nonlinear behavior is considered for each shear (friction) degree of freedom in x and y directions. The friction force-deformation relationship is given by:

$$\begin{aligned} f_x &= -P\mu_x z_x \\ f_y &= -P\mu_y z_y \end{aligned} \tag{3}$$

where f_x and f_y are the friction forces in x and y directions, μ_x and μ_y are velocity-dependent friction coefficients to consider the different coefficients of friction for fast velocity versus slow velocity conditions, and z_x and z_y are internal hysteretic variables. In order to accurately model the problem, the fast and slow friction coefficients have been considered as a function of velocity. The initial values of z_x and z_y are zero and they evolve according to following differential equation [8]:

$$\begin{Bmatrix} \dot{z}_x \\ \dot{z}_y \end{Bmatrix} = \begin{pmatrix} 1 - a_x z_x^2 & -a_y z_x z_y \\ -a_x z_x z_y & 1 - a_y z_y^2 \end{pmatrix} \begin{Bmatrix} \dfrac{K_x}{P\mu_x} \dot{d}_x \\ \dfrac{K_y}{P\mu_y} \dot{d}_y \end{Bmatrix} \qquad \text{for } \sqrt{z_x^2 + z_y^2} \le 1 \tag{4}$$

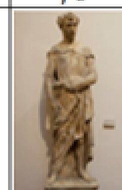
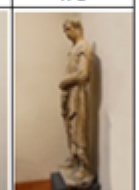
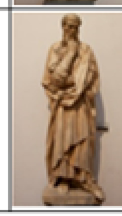
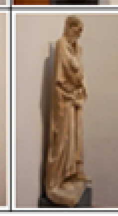
where K_x and K_y are the elastic shear stiffness constants in the absence of sliding, and a_x and a_y are binaries parameters deepening on velocity in x and y direction:

$$a_x = \begin{cases} 1 & \text{if } \dot{d}_x z_x > 0 \\ 0 & \text{otherwise} \end{cases}, \quad a_y = \begin{cases} 1 & \text{if } \dot{d}_y z_y > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

A high value of the pre-slip stiffness property in the horizontal direction has been used. This value does not affect the dynamics of the model, but only it is used to avoid ill-conditioned problem. This value has been selected as one order magnitude less than the value defined for the effective linear vertical stiffness. In addition, post slip stiffness has been set to zero (or a very small number, assigned internally), because once the friction is overcome the horizontal stiffness becomes zero.

The proposed methodology has been tested by using two real statues as case study. The two freestanding statues have been selected among the surveyed statues considered in [9]. The geometric and mass properties of the selected statues are reported in Table 1.

Table 1. Geometric and mass properties of case study statues.

Statue Name	Location	Mass [kg]	Footprint [m]		Height [m]	Center of the mass [m]			Photograph	
			x	y		x	y	z	y-z	x-z
Zuccone (Donatello)	Museo dell'Opera del Duomo	576	0.55	0.41	1.99	0.27	0.19	0.91		
Profeta (Banco)	Museo dell'Opera del Duomo	601	0.51	0.32	1.84	0.26	0.13	0.82		

Both statues have comparable mass and geometric properties and they are located in Florence, Italy. The three components (NS, EW, UD) of the Norcia earthquake in 2016, October 30th have been considered as seismic scenario. The statues have been modeled through SAP2000 according to the model represented in Fig. 2. Since the statue is considered as a rigid block, the damping has been neglected in the analyses. The stiffness property used in the horizontal directions has been fixed equal to 10 times the value used for the vertical direction for non-linear stiffness. This value is referred to the pre-slip conditions and then it is supposed to be a large number. Both fast and slow friction coefficients have been assumed equal to 0.4, whereas the rate parameter has been considered equal to 30. Thus, by applying the three selected time histories in the three directions the response in terms of center of the mass's rotational angle has been evaluated for both cases study (Fig. 3).

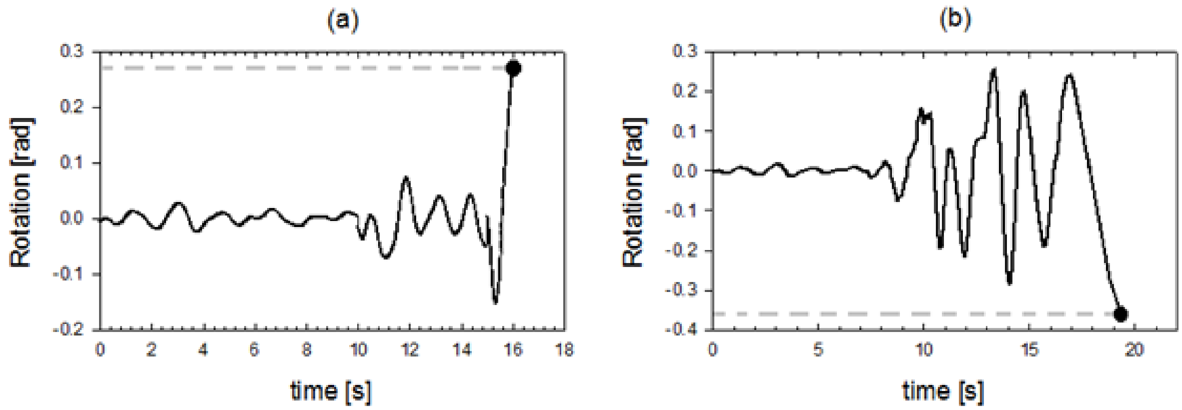


Figure 3. Rotation angle response of the first statue (a) and second one (b).

The overturning conditions can be identified by the non-convergence of the numerical direct integration of the implicit nonlinear problem. Indeed, once the rotation angle exceed the angle between the vertical axis and the line from the base edge to the center of gravity of the body at rest (α) the overturning occurs. The numerical results show how the overturning condition is identified by an uncontrolled amplification of the system's response. Fig. 3 illustrates that the overturning of the first statue (*a*) occurs before than the second one (*b*). This results is justified by the smaller dimension of the pedestal of the first statue.

Future Works

The proposed methodology will be applied to predict accurately the behavior of the heritage assets under the earthquake excitation using nonlinear SAP2000 software. Comparison with existent numerical formulations and shaking table tests will allow validating the proposed numerical approach to deepen the knowledge of the rigid bodies' dynamic in seismic conditions. It assembles finite element standard features, usually available in the literature, that can be employed to deal with the complexity of the rigid body seismic behavior, obtaining more accurate results. Indeed, different phases of motion for a 3D rigid body under a tri-directional earthquake (horizontals and vertical components) will be evaluated through the proposed methodology.

Furthermore, Monte Carlo Simulations (MCS) are applied to take into account the epistemic uncertainties associated with the geometry and mechanical properties within the range of the collected data for statues in Italy and the Mediterranean area. For each set of data, different series of geometrical parameters such as height, area of contact surface, eccentricities ratios for the center of gravity, mass, friction coefficient are considered for each model. Application Programming Interface (API) is used to assess the dynamic response of the case studies in an organized and automatic fashion. Thus, nonlinear time history analyses are performed considering different sets of earthquake intensities.

Finally, a three dimensional base isolation system is also proposed with the aim of mitigating the ground excitation. This last is performed by introducing a suitable negative stiffness device to decouple the induced vertical ground motion to the heritage asset.

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